ANALYTICAL STUDY OF HEAT AND MASS TRANSFER IN HARDENING CONCRETE DURING HEAT TREATMENT IN A CHAMBER WITH HEAT-EMITTING SURFACES

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The results of an experimental and analytical study of the heat treatment of concrete objects in chambers with heat-emitting surfaces are presented.

In the heat treatment of concretes in installations with heat-emitting surfaces [1] the walls of the chamber are heated to 200°C by induction and eddy currents and provide a source of energy for heating the mold, the concrete, and the surrounding air. Most of the heat is transferred by radiation but some undergoes convective transfer. Heat treatment is effected on the thermos principle: the apparatus is heated for 2.5-3.0 h to reach a maximum temperature and then disconnected from the supply; further heating of the contents occurs by virtue of the heat accumulated in the chamber and that evolved during the hardening of the concrete. The total duration of the cycle is 5-6 h.

The heat treatment of concrete slabs in a metal mold with an open top surface may be described by a system of heat- and mass-transfer differential equations [2] incorporating a heat source and a moisture sink:

$$\frac{\partial t(x,\ \tau)}{\partial \tau} = \frac{1}{c\gamma}\frac{\partial}{\partial x}\left[\lambda \frac{\partial t(x,\ \tau)}{\partial x}\right] + \frac{\varepsilon r}{c}\frac{\partial U(x,\ \tau)}{\partial \tau} + \frac{W}{c\gamma}, \tag{1}$$

$$\frac{\partial U(x,\ \tau)}{\partial \tau} = \frac{\partial}{\partial x}\left[a_m \frac{\partial U(x,\ \tau)}{\partial x} + a_m \delta \frac{\partial t(x,\ \tau)}{\partial x}\right] - \frac{\omega}{\gamma} \tag{2}$$

with boundary conditions

$$t(0,\ \tau) = f_1(\tau), \tag{3}$$

$$t(R,\ \tau) = f_2(\tau), \tag{4}$$

$$-\gamma a_m \left[\frac{\partial U(0,\ \tau)}{\partial x} + \delta \frac{\partial t(0,\ \tau)}{\partial x}\right] = 0, \tag{5}$$

$$-\gamma a_m \left[\frac{\partial U(R,\ \tau)}{\partial x} + \delta \frac{\partial t(R,\ \tau)}{\partial x}\right] = q_m \tag{6}$$

and initial conditions

$$t(x,\ 0) = \varphi_1(x), \tag{7}$$

$$U(x,\ 0) = \varphi_2(x). \tag{8}$$

In the boundary condition (5) the right-hand side is equal to zero, since for x = 0 there is no flow of moisture through the metal bottom of the mold, which plays the part of a moisture insulator.

An analytical solution of the system of equations (1) and (2) in the case of a nonlinear temperature variation at the surface of the concrete, with due allowance for thermal diffusion and the evaporation of

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TABLE 1. Original Data for Calculating the Temperature and Moisture Fields

Time from start of heat treatment	R, m	γ, kg/m³	t_0, °C	U_0, kg/kg	b_1, deg/h	b_2, deg/h	$a \cdot 10^4$, m²/h	$a_m \cdot 10^4$, m²/h	$\delta \cdot 10^4$, 1/deg	c, kJ/m³·deg	q_m, kg/m²·h	$\rho \cdot 10^5$, 1/h·deg
0,0	0,15	2430,0	19,0	0,0624	20	18	44,3	2,48	14,2	1510		1,4
0,5	0,15	2429,2	19,0	0,0624	20	18	40,7	4,48	7,14	2497	0,21	1,4
1,0	0,15	2428,3	19,0	0,0624	20	18	35,2	6,82	3,15	2472	0,28	1,4
1,5	0,15	2426,4	19,0	0,0624	20	18	26,1	14,90	2,05	2434	0,50	1,4
2,0	0,15	2423,9	19,0	0,0624	20	18	26,5	6,63	15,4	2397	0,71	1,4
2,5	0,15	2420,1	19,0	0,0624	20	18	32,0	5,46	17,0	2351	1,07	1,4

moisture from the surface of the object in the presence of variable thermophysical characteristics of the material, presents serious mathematical difficulties. On the basis of the specific conditions of heat treatment we shall therefore make a number of assumptions simplifying the problem.

Let us arbitrarily divide the whole period of treatment into equal time intervals. Within each interval the heat- and mass-transfer coefficients (λ, a, c, δ, a_m) and the intensity of mass outflow (q_m) will be treated as constants, while the temperature variation on the surface of the object will be regarded as linear.

Remembering the relatively low value of the thermal resistance and heat capacity of the metal mold walls, we neglect the influence of the latter on heat transfer to the object. The internal heat source and moisture sink will be regarded as uniformly distributed over the volume of the slab. Experimental investigations show that the temperature and moisture distributions over the cross section of the object may validly be treated as constant at the initial instant of the process.

Allowing for all these assumptions, the transfer equations may be written in the form

$$\frac{\partial t(x,\ \tau)}{\partial \tau} = a\frac{\partial^2 t(x,\ \tau)}{\partial x^2} + \frac{\varepsilon r}{c}\ \frac{\partial U(x,\ \tau)}{\partial \tau} + \frac{W}{c\gamma}, \tag{9}$$

$$\frac{\partial U(x,\ \tau)}{\partial \tau} = a_m\frac{\partial^2 U(x,\ \tau)}{\partial x^2} + a_m\delta\ \frac{\partial^2 t(x,\ \tau)}{\partial x^2} - \frac{\omega}{\gamma} \tag{10}$$

with boundary conditions

$$t(0,\ \tau) = t_0 + b_1\tau, \tag{11}$$

$$t(R,\ \tau) = t_0 + b_2\tau, \tag{12}$$

$$\frac{\partial U(0,\ \tau)}{\partial x} + \delta\ \frac{\partial t(0,\ \tau)}{\partial x} = 0, \tag{13}$$

$$a_m\ \frac{\partial U(R,\ \tau)}{\partial x} + a_m\delta\ \frac{\partial t(R,\ \tau)}{\partial x} + \frac{q_m}{\gamma} = 0, \tag{14}$$

$$t(x,\ 0) = t_0, \tag{15}$$

$$U(x,\ 0) = U_0. \tag{16}$$

In order to solve Eqs. (9) and (10) we must know the law of exothermic heat evolution and also the law governing the action of the internal moisture sink in the hardening concrete during heat treatment.

The rate of heat evolution depends on the thermal characteristics of the cement, the composition of the concrete, the water — cement ratio, and the time and temperature parameters of the hardening process. The exothermic behavior of cements under normal hardening conditions at 15-20°C has been extensively studied, but little attention has been paid to heat evolution in concretes during heat treatment and the dependence of the intensity of exothermic heat evolution on the hardening temperature. An analysis of work carried out in this field shows that at the present time it is quite impossible to express the cement exothermic function corresponding to the conditions of heat treatment in the form of an exact relationship, allowing for all the factors influencing heat evolution in concrete. We are thus compelled to consider only the most important of these factors and to use approximate expressions for Q_{exo}.

For the term representing the internal heat source we used the approximate formula of Voznesenskii [3]. After certain transformations this assumes the form

$$\frac{W}{c\gamma} = m[t_0 + t(x,\ \tau)], \tag{17}$$

TABLE 2. Calculated and Experimental Temperatures and Moisture Contents of Concrete during Heat Treatment

Coordinate of point x, m	Temperature t, °C, in time, h									
	0,5		1,0		1,5		2,0		2,5	
	A	B	A	B	A	B	A	B	A	B
0,015	26,1	25,5	34,5	34,3	44,6	43,8	54,0	54,2	63,9	64,2
0,045	22,1	22,0	29,8	27,8	38,8	36,5	47,2	46,8	57,6	56,5
0,075	21,3	20,5	27,8	25,7	36,3	33,0	44,5	43,3	53,2	53,2
0,105	22,1	21,5	29,0	26,7	36,7	34,7	45,8	45,0	54,4	55,3
0,135	25,4	24,0	33,1	32,7	42,4	41,5	51,0	51,3	60,0	60,0

Coordinate of point M	Moisture content U, kg/kg, in time, h									
	0,5		1,0		1,5		2,0		2,5	
	A	B	A	B	A	B	A	B	A	B
0,015	0,0615	0,0608	0,0597	0,0592	0,0558	0,0552	0,0527	0,0530	0,0480	0,0497
0,045	0,0621	0,0613	0,0616	0,0601	0,0594	0,0575	0,0572	0,0553	0,0557	0,0516
0,075	0,0621	0,0617	0,0618	0,0607	0,0604	0,0584	0,0586	0,0566	0,0577	0,0535
0,105	0,0620	0,0610	0,0615	0,0602	0,0598	0,0570	0,0573	0,0550	0,0557	0,0510
0,135	0,0610	0,0603	0,0592	0,0580	0,0572	0,0545	0,0541	0,0523	0,0500	0,0478

Note. A – calculated; B – experimental.

where

$$m = \frac{\mathrm{MCem}\sqrt{\mathrm{W/Cem}}}{c\gamma}.$$

Since the amount of combined water in the concrete is directly proportional to the amount of heat evolved in hardening [4], the term allowing for the inner moisture sink may be expressed as follows:

$$\frac{\omega}{\gamma} = p\,[t_0 + t(x, \tau)]. \tag{18}$$

Here

$$p = \frac{\mathrm{MCem}\sqrt{\mathrm{W/Cem}}}{\gamma}.$$

Allowing for (17) and (18), Eqs. (9) and (10) take the form

$$\frac{\partial t(x, \tau)}{\partial \tau} = a\frac{\partial^2 t(x, \tau)}{\partial x^2} + \frac{\varepsilon r}{c}\,\frac{\partial U(x, \tau)}{\partial \tau} + m\,[t_0 + t(x, \tau)], \tag{19}$$

$$\frac{\partial U(x, \tau)}{\partial \tau} = a_m\frac{\partial^2 U(x, \tau)}{\partial x^2} + a_m\delta\,\frac{\partial^2 t(x, \tau)}{\partial x^2} - p\,[t_0 + t(x, \tau)]. \tag{20}$$

The solution of the coupled equations (19) and (20) involves considerable difficulties. However, remembering that ε may be taken as zero (since according to Berkovich [5] moisture transfer in dense concretes takes place principally in the liquid phase during heat treatment), these equations may in practice be solved separately.

Using the operational method we obtain the following solution for the temperature field:

$$t(x, \tau) = \frac{2t_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}[(-1)^n - 1]\left[\frac{m}{A}(\exp A\tau - 1) + \exp A\tau\right]\times$$

$$\times \sin\frac{\pi n x}{e} - 2\pi a\sum_{n=1}^{\infty}\frac{n}{A^2}\{[b_1 - (-1)^n b_2](\exp A\tau - 1 - A\tau) + t_0 A\,[1 - (-1)^n](\exp A\tau - 1)\}, \tag{21}$$

where

$$A = m - \frac{\pi n^2 a}{R^2}.$$

The solution for the field of moisture contents is very complicated and laborious, and the final solution is so cumbersome that it cannot be used for engineers' calculations.

In order to obtain reasonably simple expressions suitable for calculation, we may use the asymptotic solutions of the system of equations (19) and (20) with boundary conditions (11)-(16) for short time intervals. The final equations take the form

$$t(x,\ \tau)=t_0+4\tau\sum_{n=1}^{\infty}\left\{b_1\left[i^2\operatorname{erfc}\frac{(2n-1)R-(R-x)}{2\sqrt{k\tau}}-\right.\right.$$

$$\left.-i^2\operatorname{erfc}\frac{(2n-1)R+(R-x)}{2\sqrt{k\tau}}\right]+b_2\left.\left[i^2\operatorname{erfc}\frac{(2n-1)R-x}{2\sqrt{k\tau}}-i^2\operatorname{erfc}\frac{(2n-1)R+x}{2\sqrt{k\tau}}\right]\right\}; \tag{22}$$

$$U(x,\ \tau)=U_0-2pt_0\tau-\frac{2q_m\sqrt{\tau}}{\gamma\sqrt{a_m}}\sum_{n=1}^{\infty}\left[i\operatorname{erfc}\frac{(2n-1)R-x}{2\sqrt{a_m\tau}}+i\operatorname{erfc}\frac{(2n-1)R+x}{2\sqrt{a_m\tau}}\right]-\frac{4b_1\delta\sqrt{a_m}\,\tau}{\sqrt{k}}\times \tag{23}$$

$$\times\sum_{n=1}^{\infty}\left[i^2\operatorname{erfc}\frac{(2n-1)R-x}{\sqrt{a_m\tau}}+i^2\operatorname{erfc}\frac{(2n-1)R+x}{2\sqrt{a_m\tau}}\right].$$

Here $k = a + a_m + (\varepsilon r \delta a_m / c)$.

The functions $i^n\operatorname{erfc} x$ are tabulated, and the use of solutions (22) and (23) for practical purposes presents no difficulty. Furthermore, these functions fall off rapidly with increasing argument. Hence, all terms of the series except the first are vanishingly small, and for calculation purposes we may to a sufficient accuracy confine attention to one term in the series.

Estimates of the time dependence of the heat and moisture-content fields showed that to a reasonable accuracy Eqs. (22) and (23) could be used for calculations in the period of rising temperature in the hot chamber.

The original data for the calculations are presented in Table 1, and the results are compared with experimental values of $t(x,\ \tau)$ and $U(x,\ \tau)$ in Table 2. In the experiments we used heavy concrete of composition 1 : 2 and 1 : 3.73 with a cement consumption of 335 kg/m^3 and a water/cement ratio of 0.45.

Analysis of Table 2 shows that the results of calculations based on the method presented agree quite closely with the experimental data. The resultant solution may be used for prefabricated reinforced-concrete parts such as flat slabs and panels.

NOTATION

$t(x,\ \tau)$, $U(x,\ \tau)$	are the temperature (°C) and moisture content (kg/kg), respectively, of the object at point x and time τ;
a	is the thermal diffusivity, m^2/h;
λ	is the thermal conductivity, W/m·deg;
c	is the specific heat, kJ/kg·deg;
a_m	is the moisture diffusion coefficient, m^2/h;
δ	is the thermal-gradient coefficient, 1/deg;
ε	is the phase-transition criterion;
r	is the heat of vaporization, kJ/kg;
γ	is the bulk density of concrete, kg/m^3;
W and ω	are the intensities of internal heat source ($kJ/m^3 \cdot h$) and moisture sink ($kg/m^3 \cdot h$), respectively;
R	is the characteristic dimension of the sample or object, m;
q_m	is the intensity of evaporation of moisture from the concrete, $kg/m^2 \cdot h$;
b_1 and b_2	are the rate of heating at the lower and upper surfaces of the sample, deg/h;
t_0 and U_0	are the initial temperature (°C) and moisture content (kg/kg) of the concrete;
M	is the type of cement;
Cem	is the amount of cement in the concrete, kg/m^3;
W/Cem	is the water/cement ratio of the concrete mixture.

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